

Chapter 5

Verify each identity.

1) $\cos \theta (\tan^2 \theta + 1) = \sec \theta$

$$\begin{aligned} & \cos \theta \sec^2 \theta \\ & \left(\frac{\cos}{1}\right)\left(\frac{1}{\cos^2}\right) \end{aligned}$$

↓

$\frac{1}{\cos}$
 $\sec \theta$
✓

3) $(\sin \theta + \cos \theta)^2 = 1 + \sin 2\theta$

$\sin^2 + 2\sin \cos + \cos^2$

$\sin^2 + \cos^2 + 2\sin \cos$

$$1 + 2\sin \cos \Rightarrow 1 + \sin 2\theta \quad \checkmark$$

bubble angle

Simplify Each:

5) $\left(\frac{\cot^2 \theta}{\csc \theta + 1}\right) \left(\frac{\csc - 1}{\csc - 1}\right)$

$$\rightarrow \frac{\cot^2(\csc - 1)}{\csc^2 - 1}$$

$$\Rightarrow \frac{\cot^2(\csc - 1)}{\cot^2 \csc \theta - 1}$$

6) ~~$\frac{\cos x}{\sin x + \cos x}$~~

$$\begin{aligned} & \cos x + \sin x \tan x \\ & \cos x + \left(\frac{\sin x}{1}\right) \left(\frac{\sin x}{\cos}\right) \\ & \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos} \end{aligned}$$

$$\frac{1}{\cos x} = \sec x$$

Solve each equation over the interval $[0, 2\pi]$.

8) $2\cos^2 x - \cos x = 1$

$2x^2 - x - 1 = 0$

$\tan^2 \theta + \tan \theta - 12 = 0$

$x^2 + x - 12 = 0$

$$\begin{array}{r} -12 \\ 4 \times -3 \\ \hline 1 \end{array}$$

$2\cos^2 x - \cos x - 1 = 0$

$$\begin{array}{r} x-1 \\ 2x^2-2x \\ +1x-1 \\ \hline 1 \end{array}$$

$(\tan x + 4)(\tan x - 3) = 0$

$\tan x = -4 \quad \tan x = 3$

$x = \tan^{-1}(-4) \quad x = \tan^{-1}(3)$

$$\begin{array}{l} \text{period of} \\ -\tan = 180^\circ \text{ or } \pi \end{array}$$

in degrees:

$x = 71^\circ$

$x = -76^\circ + 180^\circ$

$182^\circ + 180^\circ$

$71^\circ + 180^\circ$

$1.25^\circ + 180^\circ$

$284^\circ + 180^\circ$

$4.96^\circ + 180^\circ$

or Radians

$\{1.25, 1.82, 4.4, 4.96\}$

$0 = 2\cos x + 1 \quad \cos x - 1 = 0$

$2\cos x = -1$

$\cos x = 1$

$\cos x = -1/2$

$y = \cos^{-1}(-1/2)$

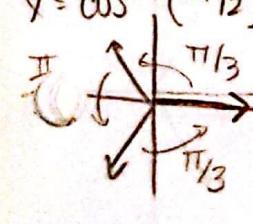
$\pi/3$

$2\pi/3$

$4\pi/3$

$2\pi/3$

$x = \{0, 2\pi/3, 4\pi/3, 2\pi\}$



Sum & diff formulas

Write the expression as the sine, cosine or tangent of an angle.

10) $\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$

$$\sin(60-45) \\ \sin(15^\circ) \approx .26$$

11) $\frac{\tan 25^\circ + \tan 10^\circ}{1 - \tan 25^\circ \tan 10^\circ}$

$$\tan(25+10) \\ \tan(35^\circ) \approx 0.7$$

Chapter 6

Use the Law of Sines or the Law of Cosines to solve the triangle. If two solutions exist, find both.

1) $B = 10^\circ, C = 30^\circ, c = 33$

Law of Sines

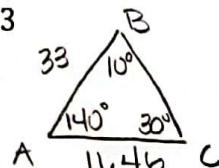
$$\frac{\sin 30}{33} = \frac{\sin 10}{b}$$

$b \sin 30 = 33 \sin 10$

$$b = \frac{33 \sin 10}{\sin 30}$$

$b = 11.46$

$\angle A = 140^\circ$



$$\frac{\sin 30}{33} = \frac{\sin 140}{a} \\ a = \frac{33 \sin 140}{\sin 30}$$

$$a = 42.42$$

2) $a = 5, b = 8, c = 10$ cosines

* Find biggest \angle 1st

$$10^2 = 8^2 + 5^2 - 2(8)(5) \cos C$$

$$100 = 89 - 80 \cos C$$

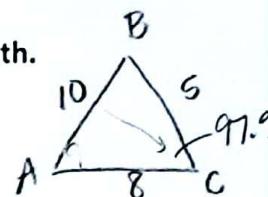
$$11 = -80 \cos C$$

$$\cos C = 11/-80$$

$$C = \cos^{-1}(11/-80)$$

$$\angle C = 97.9^\circ$$

$$\angle B = 52.4^\circ$$



3) $B = 150^\circ, a = 4, c = 4$

$$\begin{aligned} 4 &\quad 4 \\ &\quad 4 \\ &\quad 7.7 \end{aligned} \quad b^2 = 4^2 + 4^2 - 2(4)(4) \cos 150^\circ \\ b^2 = 32 - 32 \cos 150^\circ \\ b^2 = \sqrt{59.7} \quad b = 7.7$$

$$\frac{\sin A}{4} = \frac{\sin 150}{7.7}$$

$$7.7 \sin A = 4 \sin 150$$

$$A = \sin^{-1}\left(\frac{4 \sin 150}{7.7}\right)$$

$$\frac{\sin 97.9}{10} = \frac{\sin A}{5}$$

$$10 \sin A = 5 \sin 97.9$$

$$A = \sin^{-1}\left(\frac{5 \sin 97.9}{10}\right)$$

$$\angle A = 29.7^\circ$$

Find the component form, magnitude, and direction of the vector \vec{v} :

4) Initial point: $(-3, -5)$ Terminal Point: $(5, 1)$

terminal - initial

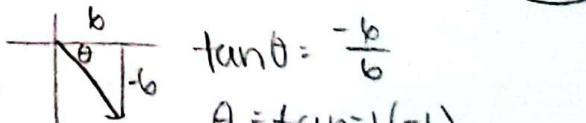
$$\langle 5+3, 1+5 \rangle$$

$$\langle 8, 6 \rangle$$

$$5) \vec{v} = 6i - 6j$$

$$\langle 6, -6 \rangle$$

$$\|\vec{v}\| = \sqrt{6^2 + (-6)^2} = \sqrt{72} = 8.5$$



$$\tan \theta = -\frac{6}{6}$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = 45^\circ \quad 360^\circ - 45^\circ \quad 315^\circ$$

Find the dot product of vectors \vec{u} and \vec{v} and determine whether they are orthogonal.

6) $\vec{u} = (6, 7); \vec{v} = (-3, 9)$

7) $\vec{u} = (8, 3); \vec{v} = (-3, 8)$

$$\vec{u} \cdot \vec{v} = 8(-3) + (3)(8)$$

$$= -24 + 24$$

$$= 0$$

Yes, orthogonal b/c
dot product = 0

$$\vec{u} \cdot \vec{v} = 6(-3) + 7(9)$$

$$= -18 + 63$$

$$= 45$$

not orthogonal

dot product $\neq 0$

the unit vector for each:

in the direction of $(2, 1)$

$$\|v\| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\frac{1}{\sqrt{5}} \langle 2, 1 \rangle = \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$$

Chapter 7

Solve the systems by substitution.

$$1) \begin{cases} x+y=2 \\ x-y=0 \end{cases} \quad x=y$$

$$\begin{aligned} y+y &= 2 & x-1 & (1, 1) \\ 2y &= 2 \\ y &= 1 \end{aligned}$$

Solve the system by elimination.

$$3) \begin{cases} 40x+30y=24 \\ 20x-50y=-14 \end{cases}$$

$$\begin{aligned} 10x+30y &= 24 & 40(x)+30(\frac{2}{5}) &= 24 \\ 10x-100y &= -28 & 40x+12 &= 24 \\ 130y &= 52 & 40x &= 12 \\ y &= \frac{2}{5} & x &= \frac{3}{10} \end{aligned}$$

$$\left\{ \frac{3}{10}, \frac{2}{5} \right\}$$

Write the form of the partial fraction decomposition of the rational expression. Do not solve for the constants.

$$5) \frac{3}{x^2+20x} = \frac{3}{x(x+20)}$$

$$\left(\frac{3}{y(y+20)} \right) = \left(\frac{A}{x} + \frac{B}{x+20} \right)^{x(x+20)}$$

$$3 = A(x+20) + B(x)$$

$$\text{let } x = -20 : 3 = 0A + -20B$$

$$B = -\frac{3}{20}$$

$$\text{let } x = 0 : 3 = 20A + 0$$

$$A = \frac{3}{20}$$

$$\left(\frac{3/20}{x} + \frac{-3/20}{x+20} \right) \text{ or } \frac{3}{20x} - \frac{3}{20(x+20)}$$

9) in the direction of $4i - 8j \quad \langle 4, -8 \rangle$

$$\|v\| = \sqrt{4^2 + (-8)^2} = \sqrt{80} = 4\sqrt{5}$$

$$\frac{1}{4\sqrt{5}} \langle 4, -8 \rangle = \left\langle \frac{4}{4\sqrt{5}}, \frac{-8}{4\sqrt{5}} \right\rangle = \left\langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \right\rangle$$

$$2) \begin{cases} x^2 + y^2 = 169 \\ 3x + 2y = 39 \end{cases} \quad 4y^2 - 78y - 78y + 1521 + 9y^2 = 1521$$

$$\begin{aligned} \frac{3x}{3} &= -\frac{2y}{3} + \frac{39}{3} \\ x &= -\frac{2}{3}y + 13 \end{aligned}$$

$$\begin{aligned} 13y^2 - 156y &= 0 \\ 13y(y-12) &= 0 \end{aligned}$$

$$\begin{aligned} y &= 0, 12 \\ x &= 13 \\ x &= 5 \end{aligned}$$

$$\{(13, 0), (5, 12)\}$$

$$4) \begin{cases} 1.5x + 2.5y = 8.5 \\ 6x + 10y = 24 \end{cases}$$

$$\begin{aligned} 6x + 10y &= 34 \\ -(6x + 10y) &= 24 \end{aligned}$$

$$0 = 10 \quad X$$

no solution

Write the form of the partial fraction decomposition of the rational expression. Do not solve for the constants.

$$6) \frac{x-8}{x^2-3x-28} \quad \left(\frac{x-8}{(x-7)(x+4)} \right) = \frac{A}{x-7} + \frac{B}{x+4}$$

$$x-8 = A(x+4) + B(x-7)$$

$$\text{let } x = -4 : -12 = 0A - 11B$$

$$B = \frac{12}{11}$$

$$\text{let } x = 7 : -1 = 11A + 0B$$

$$A = -\frac{1}{11}$$

$$\frac{-1/11}{x-7} + \frac{12/11}{x+4} \quad \text{or} \quad \left(\frac{-1}{11(x-7)} + \frac{12}{11(x+4)} \right)$$

Sketch and label the solution to the system of inequalities.

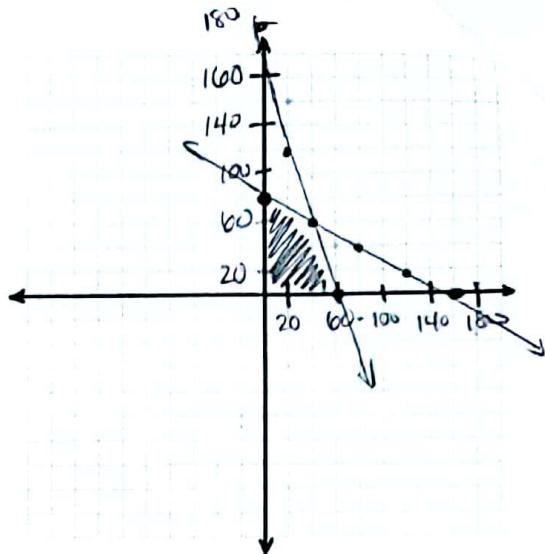
$$7) \begin{cases} x + 2y \leq 160 \\ 3x + y \leq 180 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

X	Y
0	80
160	0

$$y \leq -3x + 180$$

X	Y
0	180
60	0

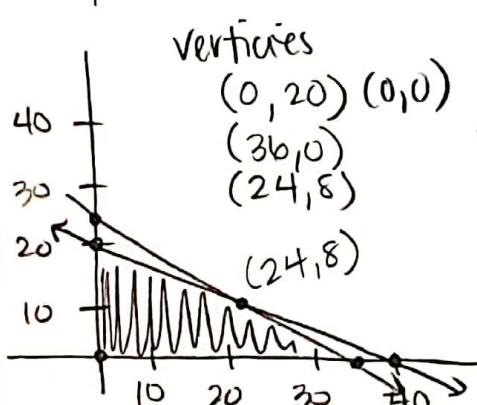
- Vertices:
- (0, 80)
 - (40, 60)
 - (60, 0)
 - (0, 0)



8) Find the maximum and minimum values of the objective function $z = 4x + y$ subject to the indicated constraints:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + 2y \leq 40 \\ 2x + 3y \geq 72 \end{cases}$$

X	Y
0	20
40	0



$$z = 4x + y$$

$$(0, 20) = 20$$

$$(36, 0) = 144$$

$$(24, 8) = 104$$

$$(0, 0) = 0$$

Chapter 8:

Min of 0
@ (0,0)
Max of 144
@ (36,0)

Given the following matrices, find the indicated information.

$$A = \begin{bmatrix} 1 & 2 \\ 5 & -4 \\ 6 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 6 & -2 & 8 \\ 4 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & -1 \\ 5 & 4 \end{bmatrix}$$

1) $A + B$

$$\begin{bmatrix} 2 & 4 \\ 3 & -3 \\ 10 & 4 \end{bmatrix}$$

2) $3C$

$$\begin{bmatrix} 18 & -6 & 24 \\ 12 & 0 & 0 \end{bmatrix}$$

3) $2A - 3B$

$$2A = \begin{bmatrix} 2 & 4 \\ 10 & -8 \\ 12 & 0 \end{bmatrix} \quad 3B = \begin{bmatrix} 3 & 6 \\ -6 & 3 \\ 12 & 12 \end{bmatrix}$$

$2A - 3B = \begin{bmatrix} -1 & -2 \\ 16 & -11 \\ 0 & -12 \end{bmatrix}$

4) $|D| \leftarrow$ determinant

$$\begin{vmatrix} 3 & -1 \\ 5 & 4 \end{vmatrix} = 3(4) - (5)(-1)$$

$$12 + 5$$

(17)

5) $D^{-1} \leftarrow$ inverse

$$\frac{1}{17} \begin{bmatrix} 4 & 1 \\ -5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4/17 & 1/17 \\ -5/17 & 3/17 \end{bmatrix}$$

6) BC

$$\begin{bmatrix} 14 & -2 & 8 \\ -8 & 4 & -16 \\ 40 & -8 & 32 \end{bmatrix}$$

Use the augmented matrix that corresponds to each system. Then solve it by writing the matrix in reduced-row echelon form.

$$7) \begin{cases} -x + y + 2z = 1 \\ 2x + 3y + z = -2 \\ 5x + 4y + 2z = 4 \end{cases} \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{array} \right]$$

$$8) \begin{cases} x + 2y + z = 6 \\ 2x + 5y + 15z = 4 \\ 3x + y + 3z = -6 \end{cases}$$

Calc!

$$\textcircled{1} \quad -2R_1 + R_2 \left[\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 5 & 5 & 0 \\ 0 & 9 & 12 & 9 \end{array} \right] \textcircled{2} \quad \frac{1}{5}R_2 \left[\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & 4 & 3 \end{array} \right]$$

$$\textcircled{3} \quad -3R_2 + R_3 \left[\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{array} \right] \textcircled{4} \quad R_1 + R_2 \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\textcircled{5} \quad R_1 + R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array} \right] \quad \{(2, -3, 3)\}$$

$$9) \text{ Evaluate } \left| \begin{array}{ccc} 10 & -5 & 5 \\ 30 & 0 & 10 \\ 0 & 10 & 1 \end{array} \right| \quad + - +$$

$$-30 \left| \begin{array}{cc} -5 & 5 \\ 10 & 1 \end{array} \right| + 0 \quad | -10 \left| \begin{array}{cc} 10 & -5 \\ 0 & 10 \end{array} \right| + - +$$

$$-30(-5 - 50) - 10(100 - 0)$$

Augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 2 & 5 & 15 & 4 \\ 3 & 1 & 3 & -6 \end{array} \right]$$

$$(-\frac{34}{13}, \frac{24}{5}, -\frac{64}{65})$$

RREF

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{34}{13} \\ 0 & 1 & 0 & \frac{24}{5} \\ 0 & 0 & 1 & -\frac{64}{65} \end{array} \right]$$

$$-30(-55) - 10(100) \\ 1650 - 1000$$

$$650$$

Solve the system using Cramer's Rule.

$$10) \begin{cases} 5x + 4y = 2 \\ -x + y = -22 \end{cases}$$

$$x = \frac{\begin{vmatrix} 2 & 4 \\ -22 & 1 \end{vmatrix}}{9} = \frac{2 + 88}{9} \Rightarrow 10$$

$$\text{denom: } \begin{vmatrix} 5 & 4 \\ -1 & 1 \end{vmatrix}$$

$$5 - (4)(-1) \\ 9$$

$$y = \frac{\begin{vmatrix} 5 & 2 \\ -1 & -22 \end{vmatrix}}{9} = \frac{-110 + 2}{9} \Rightarrow -12$$

$$(10, -12)$$

Chapter 9:

Write the first 5 terms of each sequence using the given information. Determine if the sequences are arithmetic or geometric.

$$1) a_1 = 4; d = 12 \\ \text{arithmetic}$$

$$4, 16, 28, 40, 52$$

$$2) a_1 = 25; a_k = a_{k-1} + 3 \quad \text{arithmetic}$$

$$25, 28, 31, 34, 37$$

$$3) a_1 = 2; r = \frac{1}{2} \quad \text{geometric}$$

$$2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$$

$$4) A_1 = -1; A_{k+1} = 3(A_k) \\ \text{multiply} \Rightarrow \text{Geometric}$$

$$-1, -3, -9, -27, -81$$

Write the explicit formula for the nth term for each sequence. Then, find the indicated term.

5) $5, 11, 17, 23, \dots$ 15th term

$$a_n = 5 + 6(n-1)$$

$$a_{15} = 89$$

6) $12, 6, 3, \frac{3}{2}, \dots$

$$a_n = 12 \left(\frac{1}{2}\right)^{n-1}$$

20th term

$$a_{20} = 12 \left(\frac{1}{2}\right)^{19}$$

$$a_{20} = 2.29 \times 10^{-5}$$

9) $\sum_{j=1}^{\infty} \frac{5}{10^j}$ $a_1 = \frac{1}{2}$ $a_2 = \frac{1}{20}$

∞ geometric $r = \frac{1}{10}$

$$S = \frac{y_2}{1-r} = \frac{\frac{1}{2}}{\frac{1}{10}} = \frac{1}{2} \cdot \frac{10}{1} = \frac{5}{1}$$

7) $\sum_{k=2}^8 \frac{k}{k+1}$

8) $\sum_{n=1}^{10} (2n - 3)$ Finite arithmetic

$$S_{10} = \frac{10}{2} (-1 + 17)$$

$$= 5(16)$$

$$= 80$$

10) $5, 10, 20, 40, \dots$

Infinite geometric

$$= \frac{5}{1-2} = \frac{5}{-1} = -5$$

11) $\sum_{j=1}^7 2^{j-1}$

$$\begin{array}{c} 2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6 \\ 1+2+4+8+16+32+64 \end{array}$$

$$= 127$$

or $S_n = a_1 \left(\frac{1-r^n}{1-r}\right)$

$$S_7 = 1 \left(\frac{1-2^7}{1-2}\right) = 127$$

12) $\sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^{k-1}$

Infinite geometric

$$a_1 = 1 \quad r = \frac{1}{3}$$

$$S_{\infty} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

Use the Binomial Theorem to expand the expression.

13) $(x+4)^4$

$$1(x^4)(4^0) + 4(x^3)(4^1) + 6(x^2)(4^2) + 4(x^1)(4^3) + 1(x^0)(4^4)$$

$$x^4 + 16x^3 + 96x^2 + 256x + 256$$

14) $(a-3b)^5$

$$\begin{aligned} & 1(a^5)(-3b)^0 + 5(a^4)(-3b)^1 + 10(a^3)(-3b)^2 + 10(a^2)(-3b)^3 \\ & + 5(a)(-3b)^4 + 1(a^0)(-3b)^5 \end{aligned}$$

$$a^5 - 15a^4b + 90a^3b^2 - 270a^2b^3 + 405ab^4 - 243b^5$$

Find each

15) What is the coefficient of x^4y^6 in the expansion of $(2x - 3y)^{10}$?

$$10C_6 (2x)^4 (-3y)^6$$

$$210(16x^4)(729y^6) = 2449440$$

16) What is the 4th term in the expansion of $(2x - 3y)^{10}$?

$$10C_3 (2x)^7 (-3y)^3 \quad 120(128x^7)(-27y^3) = -414720x^7y^3$$

17) A die is tossed two times. What is the probability that you will roll an even number both times?

$$2, 4, 6 = 3/6 = \frac{1}{2}$$

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

18) You have a bag of 10 blue, 4 green, and 8 red jolly ranchers. If you pick 2 jolly ranchers from the bag at random, without replacement, what is the probability that they will both be blue?

22 total

$$\frac{10}{22} \cdot \frac{9}{21} = \frac{90}{462} = \frac{13}{66} \approx 0.197$$

one less

the number of distinguishable arrangements of the letters in the word.

19) BASKETBALL

$$\frac{10!}{2! \cdot 2! \cdot 2!}$$

453,600 ways

20) HELEN

$$\frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$$

60 ways

Chapter 10

Find the indicated information.

1) Find the inclination θ of the line

$$2x - 7y + 3 = 0$$

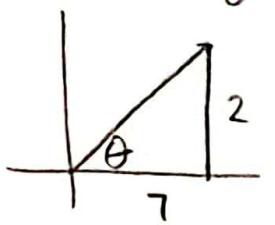
$$7y = 2x + 3$$

$$y = \frac{2}{7}x + \frac{3}{7}$$

$$\frac{2}{7} = \tan \theta$$

$$\theta = \tan^{-1}\left(\frac{2}{7}\right)$$

$$\theta = 15.9^\circ$$

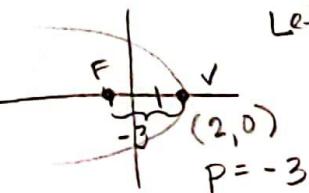


3) Find the standard form of the ~~of the~~ parabola with a vertex at $(2, 0)$ and the focus at $(-1, 0)$

Left opens $(y - k)^2 = 4p(x - h)$

$$(y - 0)^2 = 4p(x - 2)$$

$$y^2 = -12(x - 2)$$



4) Use the equation of the parabola to find the indicated information: $y^2 - 6y + 9 + 16x + 32 = 0$

a. Standard Form $(y - 3)^2 = -16(x + 2)$

$$y^2 - 6y + 16x + 41 = 0$$

b. Vertex $(-2, 3)$

$$y^2 - 6y + 9 = -16x - 41 + 9$$

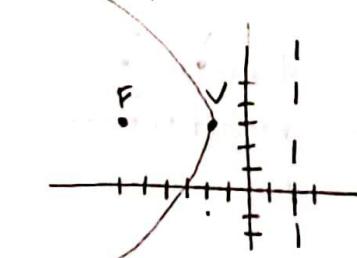
c. Focus $(-6, 3)$

$$(y - 3)^2 = -16x - 32$$

d. Directrix $x = 2$

$$(y - 3)^2 = -16(x + 2)$$

e. Sketch the parabola



5) Find the center, vertices, foci, and eccentricity of the ellipse. Then sketch it.

$$\frac{(x - 4)^2}{12} + \frac{(y + 3)^2}{16} = 1$$

$\leftrightarrow b^2$ $a^2 \downarrow$

center: $(4, -3)$

Foci: $(4, -1)$

$a^2 = 16$ $a = 4$

$(4, -5)$

$b^2 = 12$ $b = \sqrt{12}$

eccentricity: $\frac{c}{a}$

$$\frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$a^2 - b^2 = c^2$

$16 - 12 = c^2$

$4 = c^2$ $c = 2$

