Algebra II **Semester 2 Final Study Guide** **Name:** Key

Chapter 3:

Evaluate each expression, and write your answer in standard form:

1) $-2+3i+2(7-6i)$

$-2+3i+14-12i$

$12-9i$

2) $(5+6i)(-4+7i)$ FOIL $i^2=-1$

$-20+35i-24i+42i^2$

$-20+11i+42(-1) = -62+11i$

Solve each equation using any of the methods we learned this year. factoring, square roots, complete the sq or quadratic form.

3) $2x^2-17x=-30$ Box Method

$2x^2-17x+30=0$

$(2x-5)(x-6)=0$

$x=6, 5/2$

60, -5×-12, -17

	x	-6
$2x$	$2x^2$	$-12x$
-5	$-5x$	30

4) $7x^2+28=0$

$7x^2=-28$

$\sqrt{x^2}=\sqrt{-4}$

$x=\pm 2i$

5) $3x^2+13=12x$

$3x^2-12x+13=0$

$x=\frac{12\pm\sqrt{(-12)^2-4(3)(13)}}{2(3)} \rightarrow \frac{12\pm 2i\sqrt{3}}{6}$

$x=\frac{12\pm\sqrt{-12}}{6}$

$\frac{6\pm i\sqrt{3}}{3}$

6) $\frac{-2x^2-32x-34=0}{-2}$ complete square $(\frac{16}{2})^2=8^2=64$

$x^2+16x+17=0$

$x^2+16x+64=-17+64$

$\sqrt{(x+8)^2}=\sqrt{47}$

$x+8=\pm\sqrt{47}$

$x=-8\pm\sqrt{47}$

Find the discriminant of the quadratic equation and describe the number and type of solutions:

7) $-x^2-6x-9=0$

b^2-4ac

$(-6)^2-4(-1)(-9)$

$36-36=0$ 1 real soln

8) $x^2+6x=-5$

$x^2+6x+5=0$

$(6)^2-4(1)(5)$

$36-20=16$ 2 real solns

9) $x^2-2x+9=0$

$(-2)^2-4(1)(9)$

$4-36$

-32 2 imaginary solns

Solve the nonlinear system using either substitution or elimination:

9) $\begin{cases} x^2-6x+13=y \\ -y=-2x+3 \end{cases}$ $y=2x-3$

$y=2(4)-3$

$y=5$

$x^2-6x+13=2x-3$

$x^2-8x+16=0$

$(x-4)(x-4)=0$

$x=4$

16, -4×-4, -8

$(4,5)$

10) $\begin{cases} x^2+y^2=49 \\ y=7-x \end{cases}$

$x^2+(7-x)^2=49$

$x^2+(7-x)(7-x)=49$

$x^2+49-14x+x^2=49$

$2x^2-14x=0$

$2x(x-7)=0$ $x=0,7$

$y=7-0$

$y=7$

$(0,7)$

$y=7-7$

$y=0$

$(7,0)$

Solve the quadratic inequality either algebraically or graphically. Write your answer in interval notation.

11) $-x^2-10x<21$

$+x^2+10x$

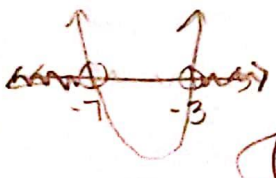
$0<x^2+10x+21$

opens up

21, 7×3, 10

$(x+7)(x+3)=0$

$x=-7,-3$



$(-\infty,-7)\cup(-3,+\infty)$

12) $3x^2+2\le 5x$

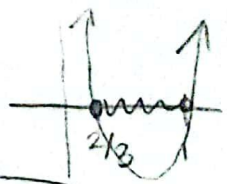
$3x^2-5x+2\le 0$

$(x-1)(3x-2)=0$

$x=1, 2/3$

6, -3×-2, -5

	$3x$	-2
x	$3x^2$	$-2x$
-1	$-3x$	2



$[2/3,1]$

Chapter 4:

Write each polynomial in standard form. Then, state the degree and leading coefficient (LC) and describe the end behavior using proper notation.

3) $g(x)=-3x^4+1-5x^2$

Standard Form: $-3x^4-5x^2+1$

Degree: 4

LC: -3

End Behavior (proper notation): $\downarrow\downarrow$

$f(x)\rightarrow -\infty$ as $x\rightarrow +\infty$

$f(x)\rightarrow -\infty$ as $x\rightarrow -\infty$

4) $f(x)=6-4x-2x^3$

Standard Form: $-2x^3-4x+6$

Degree: 3

LC: -2

End Behavior (proper notation): $\uparrow\downarrow$

$f(x)\rightarrow -\infty$ as $x\rightarrow +\infty$

$f(x)\rightarrow +\infty$ as $x\rightarrow -\infty$

Drop MAMA
Synthetic division

Simplify each expression. Write your answer in standard form:

6) $(2y^2 + 4y - 7)(y + 3)$

$2y^3+6y^2+4y^2+12y-7y-21$

$2y^3+10y^2+5y-21$

8) $(x^4 - x^2 - 7) \div (x + 4)$

$-4 \mid 1 \;\; 0 \;\; -1 \;\; 0 \;\; -7$

$\downarrow \;\; -4 \;\; 16 \;\; -60 \;\; 240$

$1 \;\; -4 \;\; 15 \;\; -60 \;\; | \;\; 233$

Lower by 1 degree

$x^3-4x^2+15x-60+\frac{233}{x+4}$

Factor Completely:

$(a\pm b)(a^2\mp ab+b^2)$

10) $128x^5 - 16x^2$

$16x^2(8x^3-1)$

$(2x)^3-(1)^3$

$16x^2(2x-1)(4x^2+2x+1)$

11) $81x^4 - 256$

$(9x^2)^2-(16)^2$

$(9x^2-16)(9x^2+16)$

$(3x+4)(3x-4)(9x^2+16)$

12) $(2x^3 - 7x^2 - 8x + 28)$

$x^2(2x-7)-4(2x-7)$

$(2x-7)(x^2-4)$

$(2x-7)(x+2)(x-2)$

13) $4x^8 - 32x^5 + 48x^2$

$4x^2(x^6-8x^3+12)$

$12 \;\; -6 \;\; -2 \;\; -8$

$4x^2(x^3-6)(x^3-2)$

14) $x^3 + 27$

$(x)^3+(3)^3$

$(x+3)(x^2-3x+9)$

Find all the real solutions and state the multiplicity of each solution:

15) $x^3 + 2x^2 = 13x - 10$

$x^3+2x^2-13x+10=0$

*graph on calc & pick x-int to start

$1 \mid 1 \;\; 2 \;\; -13 \;\; 10$

$\downarrow \;\; 1 \;\; 3 \;\; -10$

$-5 \mid 1 \;\; 3 \;\; -10 \;\; | \;\; 0$

$\downarrow \;\; -5 \;\; 10$

$1 \;\; -2 \;\; | \;\; 0$

$x-2=0$

$x=2$

$x=1, 2, -5$

all mult 1

16) $2x^4 - 9x^3 - 2x^2 + 39x - 18 = 0$

$-2 \mid 2 \;\; -9 \;\; -2 \;\; 39 \;\; -18$

$\downarrow \;\; -4 \;\; 26 \;\; -48 \;\; 18$

mult 2 b/c touches x-axis: $3 \mid 2 \;\; -13 \;\; 24 \;\; -9 \;\; | \;\; 0$

$\downarrow \;\; 6 \;\; -21 \;\; 9$

$3 \mid 2 \;\; -7 \;\; 3 \;\; | \;\; 0$

$\downarrow \;\; 6 \;\; -3$

$2 \;\; -1 \;\; | \;\; 0$

$2x-1=0$

$x=1/2$

$x = 1/2, 3, -2$

mult 2

Write a polynomial function given the following zeros:

17) $3, 2, and -5 = x$

$(x-3)(x-2)(x+5)$

$(x^2-2x-3x+6)(x+5)$

$(x^2-5x+6)(x+5)$

$x^3+5x^2-5x^2-25x+6x+30$

$x^3-19x+30=y$

18) $4, \sqrt{5}$ comes in ± pairs!

$(x+\sqrt{5})(x-\sqrt{5})(x-4)$

$(x^2-x\sqrt{5}+x\sqrt{5}-5)(x-4)$

$(x^2-5)(x-4)$

$y=x^3-4x^2-5x+20$

Use the rational root theorem to make a list of all the possible rational solutions:

Plg List!

19) $f(x) = x^3 - 8x^2 + 11x + 20$

$p: \pm 1, 2, 4, 5, 10, 20$

$q: \pm 1$

$\frac{p}{q} = \pm 1, 2, 4, 5, 10, 20$

Chapter 5:

Simplify the following expressions.

1) $\sqrt[5]{486}$

$243 \;(2)$

$9 \;\; 27$

$(3)(3) \;\; 9 \;(3)$

$(3)(3)$

$3\sqrt[5]{2}$

3) $\sqrt[4]{12} \cdot \sqrt[4]{8}$

$\sqrt[4]{96}$

$12 \;\; 8$

$6 \;(2) \;\; 4 \;(2)$

$(3)(2) \;\; (2)(2)$

$2\sqrt[4]{6}$

rationalize!

5) $\frac{1}{2-\sqrt{7}}\left(\frac{2+\sqrt{7}}{2+\sqrt{7}}\right)$

$\frac{2+\sqrt{7}}{4+2\sqrt{7}-2\sqrt{7}-7}$

$\frac{2+\sqrt{7}}{-3}$

6) $2\sqrt{48}+\sqrt{3}$

$6 \;\; 8$

$(3)(2) \;\; 4 \;(2)$

$(2)(2)$

$2(4\sqrt{3})+\sqrt{3}$

$8\sqrt{3}+\sqrt{3}$

$9\sqrt{3}$

left 2 $x+2\geq 0$, $x\geq -2$

Find the domain and range of the following. Sketch a graph if needed to help you.

7) $g(x) = -3\sqrt{x+2} - 1$

reflect over x-axis; down 1

D: $[-2, +\infty)$
R: $(-\infty, 1]$

8) $f(x) = \sqrt[3]{-x} - 6$

down 6

reflect over y-axis
* can cube root neg #'s

D: $(-\infty, +\infty)$
R: $(-\infty, +\infty)$

Solve each equation. Make sure to check for extraneous solutions.

9) $4\sqrt[3]{2x+1} - 1 = 19$

$+1 \quad +1$

$\frac{4\sqrt[3]{2x+1}}{4} = \frac{20}{4}$

$(\sqrt[3]{2x+1})^3 = (5)^3$

$2x+1 = 125$

$\frac{2x = 124}{2}$

$x = 62$

10) $x = (10x+24)^{\frac{1}{2}} + 12$

$x - 12 = \sqrt{10x+24} + 12 - 12$

$(x-12)^2 = (\sqrt{10x+24})^2$

$(x-12)(x-12) = 10x+24$

$x^2 - 24x + 144 = 10x + 24$

$x^2 - 34x + 120 = 0$

$(x-4)(x-30) = 0$ (4 × 30 = 120, −4 − 30 = −34)

$x = 4, 30$

4 doesn't work

$x = 30$

Given $f(x) = 3x - 5x^2 - x^3$ and $g(x) = 6x^2 - 4x$ find the following:

1) $(f+g)(x)$

$(3x - 5x^2 - x^3) + (6x^2 - 4x)$

$-x^3 + x^2 - x$

2) $(f-g)(-1)$

$3x - 5x^2 - x^3 - (6x^2 - 4x)$

$3x - 5x^2 - x^3 - 6x^2 + 4x$

$-x^3 - 11x^2 + 7x$

$-(-1)^3 - 11(-1)^2 + 7(-1) \Rightarrow 1 - 11 - 7 = -17$

Given $f(x) = \sqrt{x+3}$, $g(x) = 4x-3$ and $h(x) = 3x^2 + 3$, find the indicated values.

8) $(h \circ g)(x)$

$3(4x-3)^2 + 3$

$3(4x-3)(4x-3) + 3$

$3(16x^2 - 12x - 12x + 9) + 3$

$3(16x^2 - 24x + 9) + 3$

$48x^2 - 72x + 27 + 3$

$48x^2 - 72x + 30$

9) $h(f(-2))$

$f(-2) = \sqrt{-2+3}$

$= \sqrt{1} = 1$

$h(1) = 3(1)^2 + 3$

$= 3 + 3 = 6$

Find the inverse of each function. Include domain restrictions if needed.

10) $f(x) = -3x + 4$

$y = -3x + 4$

$x = -3y + 4$

$\frac{x-4}{-3} = \frac{-3y}{-3}$

$y = \frac{x-4}{-3}$

$f^{-1}(x) = \frac{x-4}{-3}$

11) $f(x) = 2\sqrt{x-3}$ D: $x \geq 3$, R: $y \geq 0$

switch x & y, solve for y

$y = 2\sqrt{x-3}$

$\frac{x}{2} = \frac{2\sqrt{y-3}}{2}$

$(\frac{x}{2})^2 = (\sqrt{y-3})^2$

$\frac{x^2}{4} = y - 3$

$+3 \quad +3$

$y = \frac{1}{4}x^2 + 3$ where $x \geq 0$

Chapter 6

Determine whether the function represents exponential growth or exponential decay.

1) $y = 2\left(\frac{1}{3}\right)^x$ — decay

2) $y = 5^x$ — $y = 1(5)^x$ — growth

3) $y = 3e^{-2x}$ neg — decay

4) $y = \frac{1}{3}e^x$ pos — growth

6) You purchased a valuable collectable for $525. The value of the collectable increases in value by 5% each year.

a) Write an exponential model in the form $y = a(b)^x$ that represents the value y (in dollars) x years after purchasing the collectable.

$y = 525(1.05)^x$

$y = a(b)^x$

1+r growth
1−r decay

b) What will be the value of the collectable after 5 years?

$y = 525(1.05)^5$

$y = \$670.05$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ or continuous compounding $A = Pe^{rt}$

7) You deposit $1,500 in an account that pays 7% annual interest. Find the balance after 2 years when the interest is compounded daily. Find the balance after 2 years when the interest is compounded continuously.

Daily: $A = 1500\left(1 + \frac{.07}{365}\right)^{(365)(2)}$

$A = \$1725.39$

Continuous: $A = 1500e^{(.07)(2)}$

$A = \$1725.41$

10) Rewrite in logarithmic form: $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$\log_{1/2}\frac{1}{16} = 4$

11) Rewrite in exponential form: $\log_7 \frac{1}{7} = -1$

$7^{-1} = \frac{1}{7}$

Use $log_7 4 \approx 0.712$ and $log_7 12 \approx 1.277$ and properties of logarithms to evaluate each logarithm.

12) $\log_7 3$

$\log_7 \frac{12}{4} = \log_7 12 - \log_7 4$

$1.277 - 0.712$

0.565

13) $\log_7 144$

$\log_7 (12)^2$

$2 \cdot \log_7 (12)$

$= 2.554$

14) $\log_7 48$

$\log_7 (4 \cdot 12)$

$= \log_7 4 + \log_7 12$

$= 1.989$

Solve the following equations. Round your answers to three decimals places if necessary.

19) $100^{5x+2} = \left(\frac{1}{10}\right)^{11-x}$ get same base

$(10^2)^{5x+2} = (10^{-1})^{11-x}$

$10^{10x+4} = 10^{-11+x}$

$10x + 4 = -11 + x$

$-x \quad -4 \quad -4 \quad -x$

$9x = -15$

$x = -5/3$

20) $\log_6 3x + \log_6 (x-1) = 3$ condense

$\log_6 3x(x-1) = 3$

$\log_6 3x^2 - 3x = 3$

$6^3 = 3x^2 - 3x$

$3x^2 - 3x - 216 = 0$

$3(x^2 - x - 72) = 0$

$3(x+8)(x-9) = 0$

$x = -8, 9$

$x = 9$

21) $6 = \log_3(2x+1) + 4$

$-4 \quad -4$

$2 = \log_3 (2x+1)$

$3^2 = 2x + 1$

$9 = 2x + 1$

$8 = 2x$

$x = 4$

22) $2e^{2x} - 7 = 5$

$+7 \quad +7$

$\frac{2e^{2x}}{2} = \frac{12}{2}$

$e^{2x} = 6$

$\ln e^{2x} = \ln 6$

$2x \ln e = \ln 6$

$x = 0.896$

23) Write an exponential function $y = a(b)^x$ whose graph passes through $(0, 100)$ and $\left(4, \frac{25}{4}\right)$

$(x, y) \quad (x, y)$

$100 = a(b)^0$

$100 = a$

$\frac{25}{4} = 100(b)^4$

$\frac{1}{100} = \frac{100(b)^4}{100}$

$\sqrt[4]{1/16} = \sqrt[4]{b^4}$

$b = \frac{1}{2}$

$y = 100\left(\frac{1}{2}\right)^x$

Chapter 7

Simplify each expression. State the excluded values.

1) $\frac{-4ab}{21c} \cdot \frac{14c^2}{22a^2}$

$= \frac{(-4)(14)abc^2}{(21)(22)a^2c}$

$= \frac{-4bc}{33a}$

$a, c \neq 0$

2) $\frac{x-3}{2x-8} \cdot \frac{6x^2-96}{x^2-9}$

$6(x^2-16) \Rightarrow 6(x+4)(x-4)$

$\frac{x-3}{2(x-4)} \cdot \frac{6(x+4)(x-4)}{(x+3)(x-3)} = \frac{3(x+4)}{(x+3)}$

$x \neq \pm 3, 4$

3) $\frac{y^2-y-12}{y+2} \div \frac{y-4}{y^2-4y-12}$

$\frac{(y-4)(y+3)}{(y+2)} \div \frac{(y-4)}{(y-6)(y+2)}$

$y \neq -2, 6, 4$

$\frac{(y-4)(y+3)}{(y+2)} \cdot \frac{(y-6)(y+2)}{(y-4)}$

$(y+3)(y-6)$

$y^2 - 3y - 18$

4) $\left(\frac{x-2}{x-1}\right) - \frac{6}{7x-7}$ need common denom LCD: $7(x-1)$

$7(x-1)$

$\frac{7x-14}{7(x-1)} - \frac{6}{7(x-1)} = \frac{7x-20}{7(x-1)}$

$x \neq 1$

Solve each equation or inequality: get rid of den. LCD: $x(x+4)$

7) $\frac{x}{x+4} = \frac{16}{x^2+4x} + \frac{2}{x}$

$x(x+4)\left(\frac{x}{x+4}\right) = \left(\frac{16}{x(x+4)}\right)x(x+4) + \left(\frac{2}{x}\right)x(x+4)$

$x^2 = 16 + 2(x+4)$

$x^2 = 16 + 2x + 8$

$x^2 - 2x - 24 = 0$

-24: $-6 \cdot 4$, -2

$(x-6)(x+4) = 0$

$x = 6, -4$

but $x \neq -4$

$x = 6$

Complete the following application problems.

8) The time it takes to travel from North Vancouver to Hope varies inversely as the speed at which one travels. If it takes 1.5 hours to travel this distance at an average speed of 120 km/h, find the amount of time it would take to drive back if you were only able to travel at 60 km/h due to an engine problem.

$y = \frac{k}{x} \Rightarrow t = \frac{k}{s}$ $\quad 1.5 = \frac{k}{120}$ $\quad k = 180$ mi $\quad t = \frac{180}{60}$

$t = 3$ hours

9) When looking at two buildings at the same time, the length of the buildings' shadows varies directly as their height. If a 5-story building has a 20 m long shadow, how many stories high would a building that has a 32 m long shadow be?

$y = kx$ $\quad$ shad $= k \cdot$ height

$20 = k(5)$

$k = 4$

shad $= 4 \cdot$ height

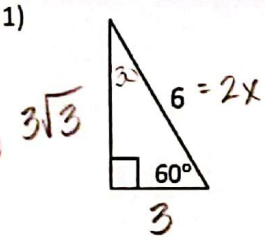
$32 = 4 \cdot h$

$h = 8$ stories

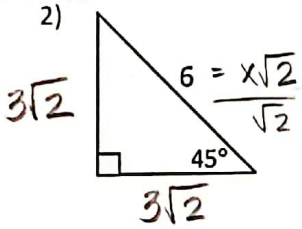
Chapter 10.1-10.3

Find all the missing sides of each special right triangle:

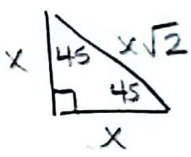
1)

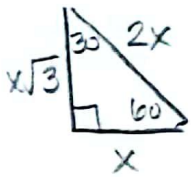


2)



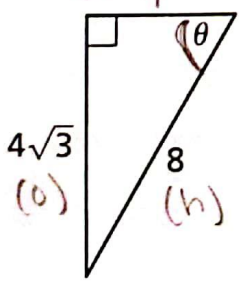
$x = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$





4) **Evaluate the six trig functions of θ. Make sure to simplify your answers.** SOH CAH TOA

(a) 4



$(4\sqrt{3})^2 + b^2 = 8^2$

$16(3) + b^2 = 64$

$48 + b^2 = 64$

$b^2 = 16$

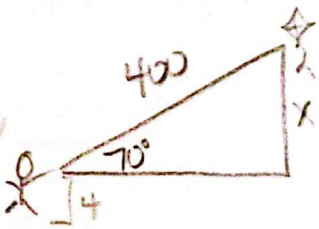
$b = 4$

$\sin\theta = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $\quad \csc\theta = \frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{4}{8} = \frac{1}{2}$ $\quad \sec\theta = 2$

$\tan\theta = \frac{4\sqrt{3}}{4} = \sqrt{3}$ $\quad \cot\theta = \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{3}}{3}$

6) **You fly a kite at an angle of 70°. The length of the string is 400 feet, and you hold the reel 4 feet above the ground. How high above the ground is the kite?** SOH CAH TOA



$\sin 70 = \frac{x}{400}$

$400 \sin 70 = x$

$x = 375.88$

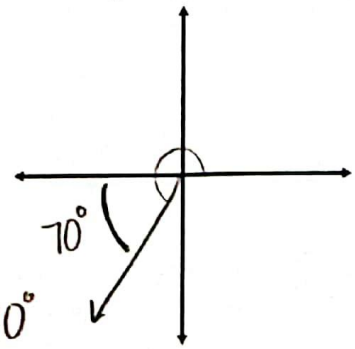
$+ 4$

379.9

$x = 379.9$ feet

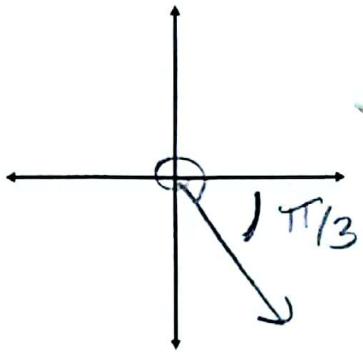
Draw each angle in standard position. Then, find and label the reference angle for each.

7) 250°



Reference Angle: 70°

8) $-\frac{7\pi}{3}$ $-2\frac{1}{3}\pi$



Reference Angle: π/3

9) Find one positive angle and one negative angle that are coterminal to 25°

pos: 25+360 = 385°
neg: 25−360 = −335°

Convert the degree measure to radians, or the radian measure to degrees:

10) $\frac{7\pi}{2}$ to degrees

$\frac{7\pi}{2} \mid \frac{180}{\pi}$ (90) = 630°

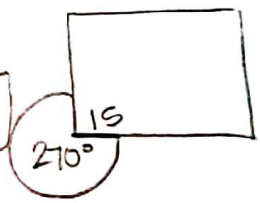
11) −425° to radians

$\frac{-425^\circ \mid \pi}{180^\circ} = \frac{-425\pi}{180} = \frac{-85\pi}{36}$

10) A dog is leashed to the corner of a rectangular house. The leash is 15 feet long and the dog can run 270° from the point that the leash is connected to the corner of the house.

a) How much running area does the dog have? $A = \frac{\theta}{360}\pi r^2$

$A = \frac{270}{360}\pi(15)^2 = \frac{3}{4}\cdot\frac{225\pi}{1} =$ 530.1 ft²



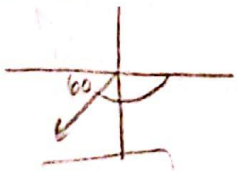
b) You want to put fencing along the perimeter of the dog's running area. You do not put fencing along the sides of the house. How many feet of fencing do you need? Arc length = $\frac{\theta}{360}(2\pi r)$

$S = \frac{270}{360}(2\pi(15)) = \frac{3}{4}\left(\frac{30\pi}{1}\right) =$ 70.7 ft

Use your unit circle to find the exact values of the following (no decimals, use your unit circle!) (cos, sin)

11) cos(−120°)

$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$



$\cos(-120) = -\frac{1}{2}$

12) tan 330°

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\tan = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$-\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

15) $\sin\frac{8\pi}{3} = 2\frac{2}{3}\pi$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{3}}{2}$

***You will need to look over the probability and two-way table notes from Chapter 8 as there will be questions about them on the final!**